Time evolution of correlations in strongly interacting fermions after a quantum quench

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Using the adaptive time-dependent density matrix renormalization group, we study the time evolution of density correlations of interacting spinless fermions on a one-dimensional lattice after a sudden change in the interaction strength. Over a broad range of model parameters, the correlation function exhibits a characteristic light-cone-like time evolution representative of a ballistic transport of information. Such behavior is observed both when quenching an insulator into the metallic region and also when quenching within the insulating region. However, when a metallic state beyond the quantum critical point is quenched deep into the insulating regime, no indication for ballistic transport is observed. Instead, stable domain walls in the density correlations emerge during the time evolution, consistent with the predictions of the Kibble-Zurek mechanism.

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I. INTRODUCTION

New experimental possibilities to manipulate ultracold atoms with a high degree of control[1,2,3,4] have led to a renewed interest in the dynamics of quantum many-body systems out of equilibrium. In these systems, the coupling to the environment is negligible, so that perturbation by external couplings, which would cause relaxation, does not influence their time evolution. A particularly simple realization of a time-dependent perturbation is a so-called quantum quench, in which a nonequilibrium situation is induced by suddenly changing one or more parameters of the system, such as the interaction strength between the particles. In bosonic systems, experiments have been realized in which a collapse and revival of the initial state is observed,[1] as well as others in which relaxation to non-thermal states in one dimension,[2] where the system becomes virtually integrable, takes place. Of particular interest are quenches through a quantum critical point.[3,4] When such quenches are fast enough and end in a phase characterized by spontaneously broken symmetry, Kibble[5] and Zurek[6,7] have predicted the spontaneous formation of topological defects. In this scenario, domains with different realizations of the possible vacua of the broken symmetry state are created, giving rise to topological defects.

Recent theoretical work has investigated the nature of quasi-stationary states in correlated quantum systems reached a sufficiently long time after quenches.[8,9,10,11,12,13] In addition to studying the long-time behavior, it is also of interest to provide a physical picture for the mechanism of the evolution by investigating the short-time behavior of relevant correlation functions. In the case where the quench is performed at a critical point, Calabrese and Cardy[14,15] have provided a general picture based on renormalization group arguments and, in one dimension, on boundary conformal field theory. In particular, for a one-dimensional system which possesses quasiparticle excitations with a typical velocity v, they predict the formation of a light cone, i.e., correlations between points at distance x are established after time $t \sim x/2v$. This general picture has been numerically confirmed recently in the Bose-Hubbard model using the adaptive time-dependent density matrix renormalization group (t-DMRG)[16,17,18] and exact diagonalization techniques.[19] This system is known to undergo a quantum phase transition from a superfluid to a Mott-insulating phase. The numerical results indicate that a horizon is created irrespective of the criticality of the system, confirming the picture of Calabrese and Cardy that the ballistic transport of quasiparticle excitations is the leading contribution to information transfer in a generic quench.

Here we study a system of spinless fermions on a one-dimensional lattice at half filling with a nearest-neighbor repulsion V using the t-DMRG method. This model is equivalent to the anisotropic Heisenberg (XXZ) chain and can be obtained from it by applying a Jordan-Wigner transformation.[20] At $V_c = 2t_{\mathrm{h}}$, where t_{h} is the nearest-neighbor hopping amplitude, the ground-state phase diagram of the model contains a quantum critical point separating a Luttinger liquid phase ($V < V_c$) from a charge-density-wave (CDW) insulator. The CDW phase corresponds to a spontaneous breaking of lattice translational symmetry through the doubling of the unit cell, so that there are two degenerate sectors for the ground state. Hence, for a sufficiently rapid quench into the CDW phase, domain walls might be expected to be created, as predicted by the Kibble-Zurek mechanism. We carry out quenches starting with initial states in either phase and suddenly change V to a value that corresponds to another point in the same phase, to the other phase, or to the critical point. This will allow us to study the extent to which the picture of Calabrese and Cardy is valid, as well as the details of the evolution of the density correlation function in a regime that goes beyond the scaling

theory of the Kibble-Zurek mechanism. In addition, we consider the time evolution of the von Neumann entropy of subsystems of varying size, which provides a measure of the spread of entanglement in the system.[21,22,23,24]

The paper is organized as follows. In Sec. II, we describe the model, how the quench is carried out, and some details of the calculations. In Sec. III, we present our results for the density correlation functions and the block entropy for the various quench scenarios. In Sec. IV we summarize and discuss our findings.

II. MODEL AND METHOD

In the following, we consider interacting spinless fermions on a one-dimensional lattice at half filling, described by the Hamiltonian

$$\hat{H} = -t_{\mathrm{h}} \sum_j \left(c^\dagger_{j+1} c_j + h.c. \right) + V \sum_j n_j n_{j+1} \, , \qquad (1)$$

with nearest-neighbor hopping amplitude t_{h} and nearest-neighbor repulsion V. The operator $c_i^{(\dagger)}$ annihilates (creates) a fermion at lattice site i, and $n_i = c_i^\dagger c_i$ is the local density operator at site i. In the following, we take $\hbar = 1$, set the lattice constant $a = 1$, and measure energies in units of t_{h} and times in units of $1/t_{\mathrm{h}}$. We denote the above Hamiltonian for a given value of V as $H(V)$. The ground-state phase diagram of the integrable quantum system described by $H(V)$ can be obtained via the Bethe ansatz[25] and is well known to contain a quantum phase transition at $V_c = 2$ from a Luttinger-liquid regime for $V < V_c$ to a charge-density-wave (CDW) insulating regime for $V > V_c$, in which translational symmetry is spontaneously broken in the thermodynamic limit. We are interested here in the time evolution after a quantum quench in which the initial state of the system is prepared in the ground state of the Hamiltonian $H(V_0)$ with an initial value of the interaction set to $V_0 = 0.5$ or $V_0 = 10$, i.e., we consider quenches starting from both a metallic state and an insulating one. After the quench, i.e., the sudden change of the interaction parameter, the dynamics of the closed quantum system is governed by the Schrödinger equation with Hamiltonian $H(V)$, with a value $V \neq V_0$. Quenches that cross the quantum critical point will be treated, as well as those that remain within the same phase. We calculate the time evolution of the equal-time density correlation function

$$C_{i,j}(t) = \langle n_i(t) n_j(t) \rangle - \langle n_i(t) \rangle \langle n_j(t) \rangle \, , \qquad (2)$$

as well as the local density $\langle n_i(t) \rangle$ of the system. The time evolution of these observables is computed using the adaptive t-DMRG for systems with $L = 49$ and, in some cases, $L = 50$ lattice sites, both with $N = 25$ particles. In choosing an odd number of lattice sites, we ensure a unique ground state in the CDW regime, accessible to the DMRG method. The results presented for $C_{i,j}(t)$ are computed from the center of the system. We confirm explicitly that reflection symmetry remains conserved over the time periods considered. We perform the calculations by fixing the discarded weight to 10^{-9}, keeping up to several hundred states during the time evolution. In the following, we concentrate on the behavior of the system for times $t \leq 5$, whereas in a previous publication,[13] we focused on the long-time relaxation behavior after a quantum quench in this system.

As an additional probe for the propagation of information through the system after the quench, we calculate within the t-DMRG the time evolution of the von Neumann entropy for a subsystem A of l sites,

$$S_l(t) = -\mathrm{Tr} \left[\varrho_A \log(\varrho_A)(t) \right] , \qquad (3)$$

where $\varrho_A(t)$ is the reduced density matrix of the subsystem.[26] The von Neumann entropy in Eq. (3) is a measure of the entanglement of subsystem A with the remainder of the system. Its time evolution therefore enables us to quantify the growth of the entanglement in the quantum system after the quench.[21,22,23,24] In the following sections, we will discuss the propagation of information and entanglement after the quantum quench based on the time evolution of the quantities described above.

III. RESULTS

A. CDW initial state

We begin the discussion of our results by considering quenches that start from a CDW ground state, i.e., $H(V_0)$ with $V_0 = 10$. In Fig. 1, we provide a two-dimensional representation of the time evolution of the equal-time density correlation function $C_{i,j}(t)$ for $V = 0$, 2, 5, and 20, corresponding to Hamiltonians $H(V)$ with (noninteracting) Luttinger liquid ($V = 0$), quantum critical ($V = 2$), or CDW ($V = 5$, 20) ground states, respectively. At time $t = 0$, the density correlations decay exponentially with the separation $|i - j|$ due to the charge gap of the CDW ground state. After the quench, we observe that the time evolution of $C_{i,j}(t)$ exhibits a pronounced signature of a light cone in all cases. The local densities at two separated lattice sites remain essentially uncorrelated up to a time t_d, which increases linearly with their spatial separation $d = |i - j|$. The corresponding propagation velocity u of the front can be obtained from the slope of the light cone using $t_d = d/u$. For systems describable by conformal field theory and for various exactly solvable models, both in the continuum and on discrete lattices, Calabrese and Cardy have shown that the velocity of this light-cone-like spread of the correlations is twice the maximal group velocity v of the fastest excitations. The relevant excitations in the present model of spinless fermions are the sound modes of the local density fluctuations. In particular, for the free case $V = 0$, the slope of the first front corresponds

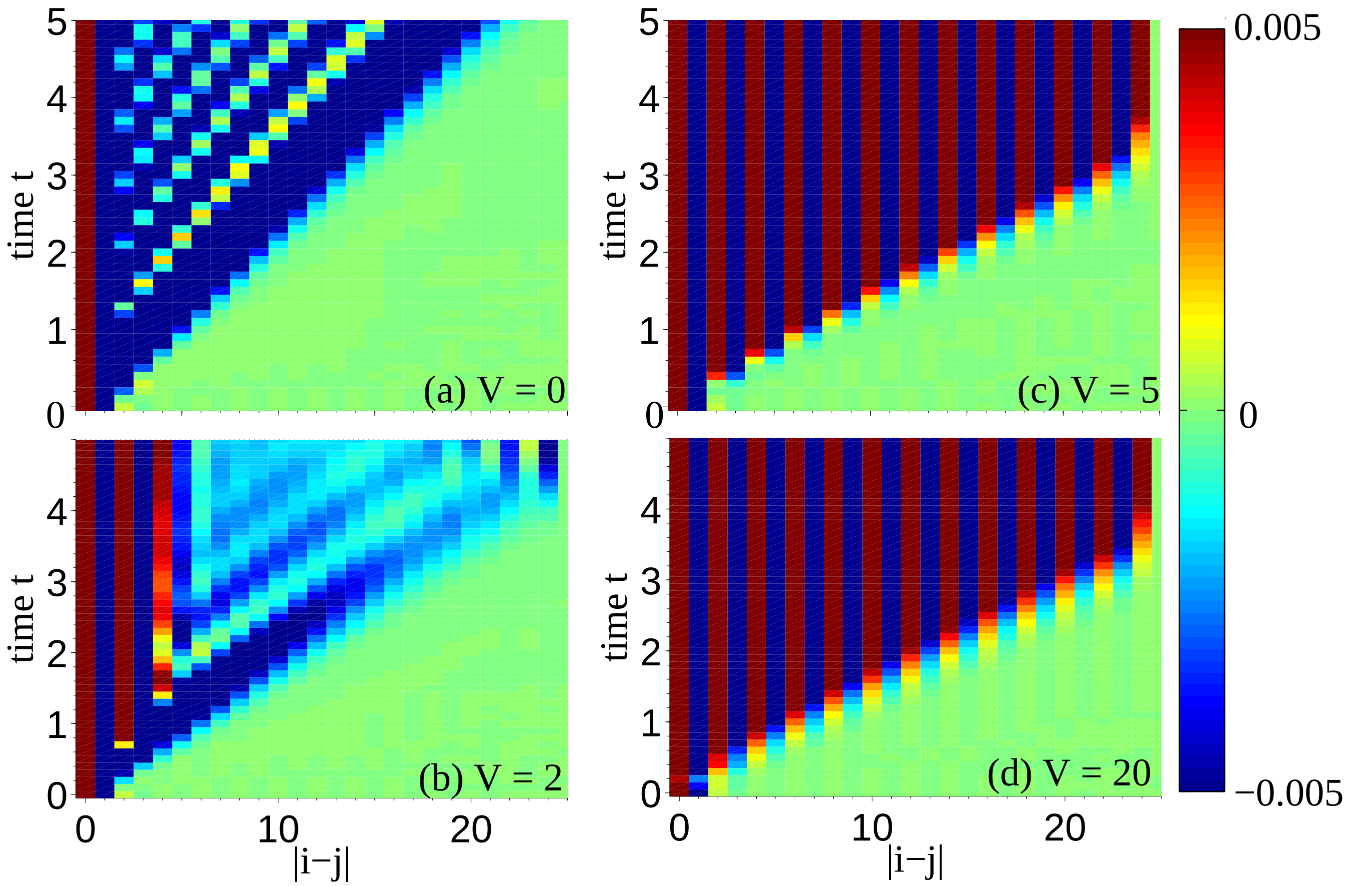

FIG. 1: (color online) Time evolution of the equal-time density correlation function $C_{i,j}(t)$ of spinless fermions after a quench from the CDW ground state of $H(V_0)$ with $V_0 = 10$, evolved by the Hamiltonian $H(V)$, with (a) $V = 0$, (b) $V = 2$, (c) $V = 5$, (d) and $V = 20$.

to $u(V = 0) = 2v_F = 4t_{\mathrm{h}}$, as expected, where v_F denotes the Fermi velocity for $V = 0$. In addition to the light cone, additional propagation fronts at later times can be identified in Fig. 1(a), which, however, possess a lower velocity. This signals that slower quasiparticles stemming from regions without linear dispersion also participate in spreading information. Figure 1(c) shows the evolution of the correlation function for a quench within the CDW phase, i.e., a case which should not be describable by conformal field theory. Interestingly, we nevertheless find a pronounced light-cone behavior in the correlation function. Although the conformal field theory underlying the treatment of Calabrese and Cardy is not valid in this region, the physical picture that ballistically propagating quasiparticles are generated by the quench seems to hold. However, in contrast to the case of the quench to the LL displayed in Figs. 1(a) and (b), we see that a strong alternating pattern forms in the density correlation function and remains present and qualitatively unchanged after the onset of the light cone.

A more detailed view of the temporal evolution of the correlation functions is shown in Fig. 2, in which we plot the values of $C_{i,j}(t)$ as a function of time for increasing distance $\mid i-j \mid$ for $V = 0$ and $V = 2$, the two extremes of the Luttinger-liquid phase. After the arrival of the first signal, oscillatory behavior as a function of time can be observed at each distance. However, as V is increased, the observed oscillations both decrease in magnitude and are damped out more rapidly. Comparing the results for the free case to the ones obtained for $V = 2$ in Fig. 2, it can be seen that the incoming front travels with a higher velocity when V is larger, as can also be seen in Fig. 1.

In contrast to the oscillatory behavior in the Luttinger-liquid phase, a steady increase of the correlations is observed when the quench occurs within the CDW phase, as can be seen in Fig. 3. The alternating pattern imprinted at the onset of the light cone is preserved. Presumably, the correlation functions saturate at some time that is significantly longer than the maximum time reached here. While results for both $V < V_0$ and $V > V_0$ show the same

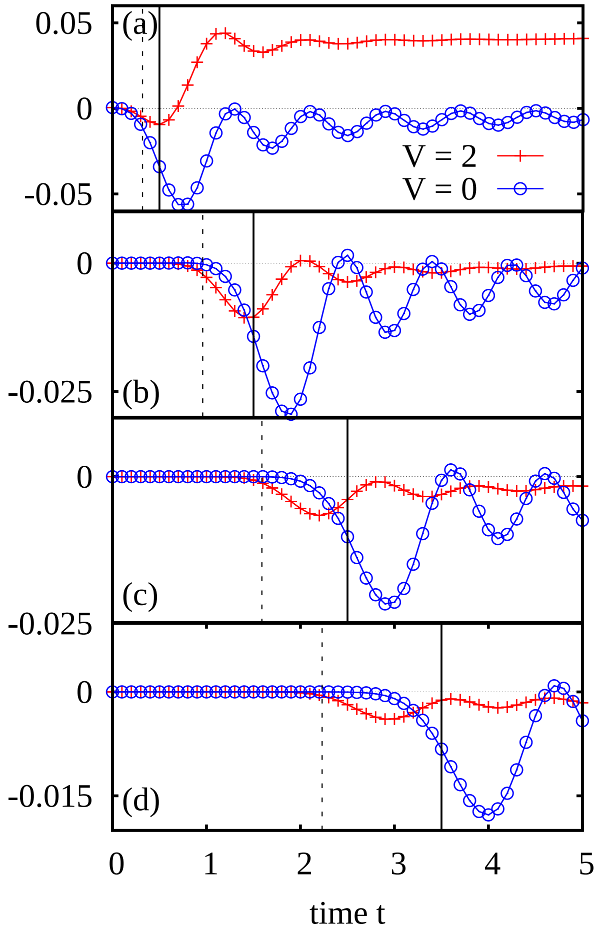

FIG. 2: (color online) Equal-time density correlation function $C_{i,j}(t)$ of spinless fermions after a quench from the CDW ground state of $H(V_0)$ with $V_0 = 10$, evolved by the Hamiltonian $H(V)$, with $V = 0$ and $V = 2$, at fixed separations (a) $|i-j| = 2$, (b) 6, (c) 10, and (d) 14 as functions of time t. The black vertical lines indicate the position of the horizon for $V = 0$, which moves with velocity $u = 4$, and the dashed vertical lines a horizon moving with velocity $u = 2v(V = 2) = 2\pi$.

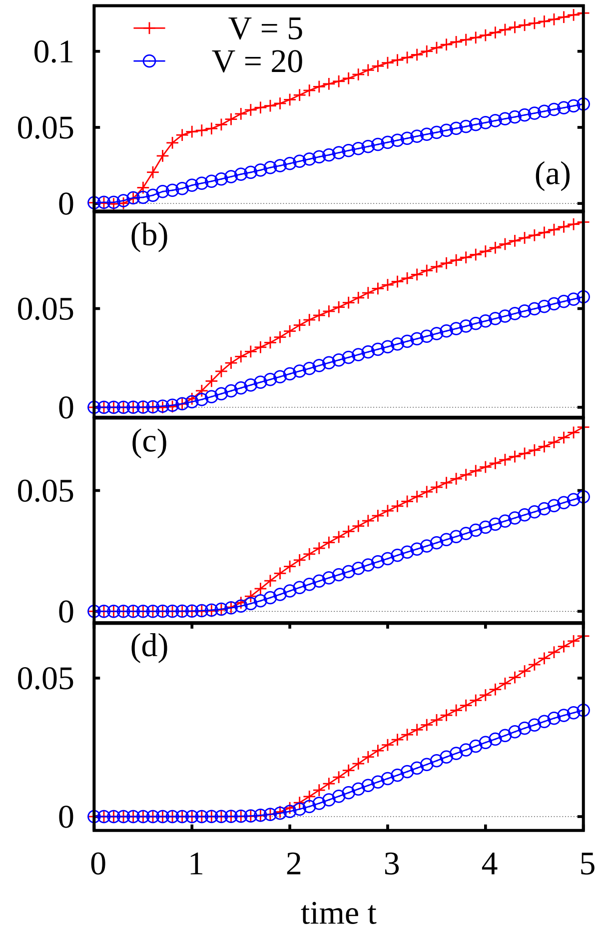

FIG. 3: (color online) Equal-time density correlation function $C_{i,j}(t)$ of spinless fermions after a quench from the CDW ground state of $H(V_0)$ with $V_0 = 10$, evolved by the Hamiltonian $H(V)$, with $V = 5$ and $V = 20$, at fixed separations $|i-j| = 2$ (a), 6 (b), 10 (c), and 14 (d) as functions of time t after the quench.

qualitative behavior, a difference is observed in the rate and the form of the increase of the correlation functions. The increase of the data is stronger for $V = 5$ in Fig. 3 but is sublinear, while the increase is linear to a very good approximation for $V = 20$.

This increase of correlations is remarkable. The length over which the correlation function falls off is much larger than the initial correlation length after the quench, as can be seen in Fig. 4 by comparing the initial $C_{i,j}(t)$ [Fig. 4 (a)] with the results for later times. This is analyzed in more detail in Fig. 5 for the quench from $V_0 = 10$ to $V = 5$. As can be seen on the semilogarithmic scale, $|C_{i,j}(t)|$ decays exponentially for intermediate distances at all times $t \leq 5$. The correlation length and the distance over which the decay is exponential both increase substantially with time. This can be contrasted to the results of Ref. 9 for the Luttinger model where, starting from a gapless phase, an algebraic decay of the correlations is found.

Note that the characteristic velocity of the light cone increases as V is changed from $V = 0$ to $V = 2$, as can be seen in Figs. 1 (a) and (b) and in Fig. 2, but is approximately the same for $V = 5$ and $V = 20$, Figs. 1 (c) and (d) and Fig. 3. The increase of velocity in the Luttinger liquid phase can be understood on the basis of the Bethe ansatz solution of the equivalent spin-1/2 XXZ model,[25] where the relevant maximum mode velocity in the Luttinger liquid regime $V < 2$ can be obtained as[27]

$$v(V) = v_F \frac{\pi\sqrt{1-(V/2)^2}}{2\arccos(V/2)}, \qquad (4)$$

where in our case $v_F = 2t_{\rm h}$. It can be seen that the front

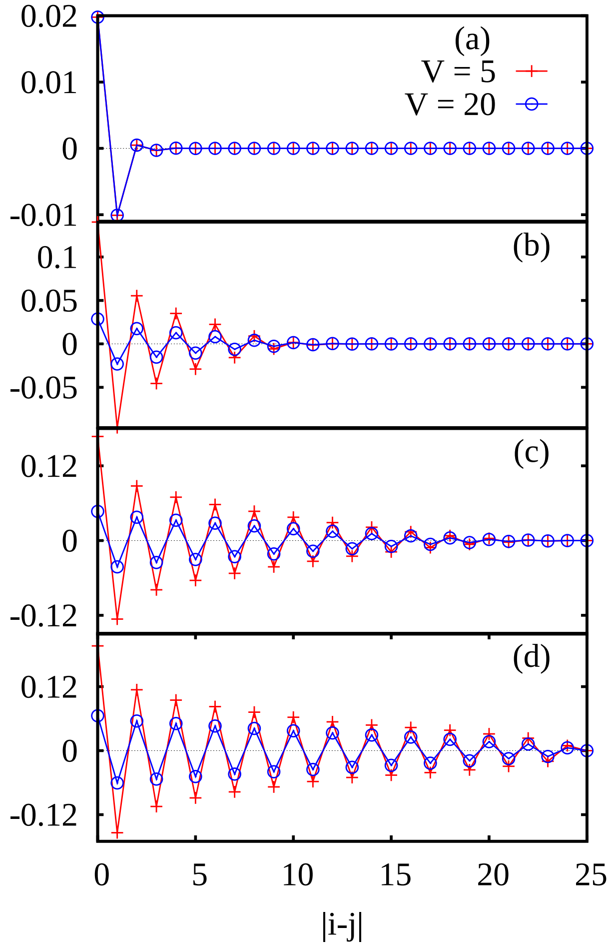

FIG. 4: (color online) Equal-time density correlation function $C_{i,j}(t)$ after a quench from the CDW ground state of $H(V_0)$ with $V_0 = 10$, evolved by the Hamiltonian $H(V)$, with $V = 5$ and $V = 20$, at fixed times (a) $t = 0$, (b) $t = 1.4$, (c) $t = 2.8$, and (d) $t = 4.2$ (d) as functions of the separation $|i - j|$.

propagating with velocity $u = 2v(V)$ indeed coincides with the shoulder of the first peak in the signal of $C_{i,j}(t)$ in Fig. 2. We display the light-cone velocities for different values of V in Fig. 6, together with the result from the Bethe ansatz. We see that inside the Luttinger liquid regime, $V < 2$, the horizon travels at a velocity $u(V)$ that approximately follows the Bethe-ansatz prediction $2v(V)$, but is consistently somewhat smaller, except at $V = 0$. We thus confirm the general picture obtained by Calabrese and Cardy of ballistic propagation of information through the system, but find that the propagation does not take place with the maximal possible velocity. On general grounds, conformal field theory becomes exact in the asymptotic limit of a continuum theory in which the quasiparticles have a linear dispersion relation. However, after the quench, quasiparticles are excited over a broad energy range and hence generally have velocities lower than the maximum set by Eq. 4.

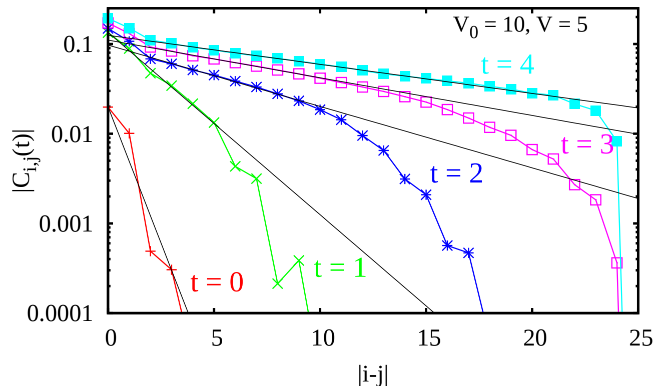

FIG. 5: (color online) Equal-time density correlation function as a function of distance $|i-j|$ at different times for the quench from $V_0 = 10$ to $V = 5$. The black lines are fits to exponential decays.

We attribute the reduced apparent light-cone propagation velocity found in Fig. 6 to the contribution of these lower-velocity quasiparticles.

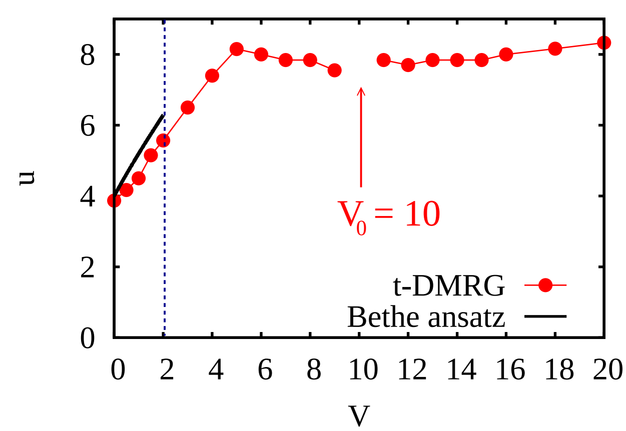

FIG. 6: (color online) Velocity of the the horizon in the time evolution of the equal-time density correlation function $C_{i,j}(t)$ of spinless fermions after a quench from the CDW ground state of $H(V_0)$ with $V_0 = 10$, evolved by the Hamiltonian $H(V)$ as a function of V. The estimated error is of the order of the point size. The dashed vertical line indicates the quantum phase transition form the Luttinger liquid to the CDW regime.

For values of the interaction beyond the quantum critical point, the velocity of the horizon can still be determined, as is evident in Figs. 1 (c) and (d). This velocity continues to increase up to $V \approx 5$. Beyond this value, $u(V)$ remains essentially constant, as could already be seen in Fig. 3.

We conclude this section by comparing the commensurability of the correlations after quenches that remain in the CDW phase to that of ones quenching to the LL phase. The presence of a CDW ground state for $V_0 = 10$ leads to strong commensurate spatial oscilla-

tions in $C_{i,j}(t)$ which increase in time, as demonstrated for $V = 5$ and $V = 20$ in Fig. 4. For quenches beyond the insulating regime, however, we also observe spatial oscillations in $C_{i,j}(t)$, but with larger spatial periods incommensurate to the underlying lattice (see Fig. 1). In the propagating quasiparticle picture of Calabrese and Cardy, such spatial oscillations are expected to arise in the time evolution of correlation functions as a generic property of a system with a nonlinear dispersion relation. It is rather remarkable that for interaction quenches that stay within the CDW regime, the time evolution leads to the buildup of extended commensurate correlations even though the ground states in this insulating regime possess a very fast exponential decay of the very same correlation function.

B. Luttinger-liquid initial state

We now consider quenches that start from the Luttinger-liquid ground state of $H(V_0)$, taking $V_0 = 0.5$. We focus particularly on quenches that go beyond the quantum critical point to the symmetry-broken CDW regime. In Fig. 7, we show the time evolution of the equal-time density correlation function $C_{i,j}(t)$ in a two-dimensional representation for $V = 5$ and $V = 40$. At $t = 0$, the density correlations $C_{i,j}(0)$ decay algebraically due to the critical nature of the $V_0 = 0.5$ ground state. For quenches with $V \lesssim 5$, a light cone can be identified, as seen in Fig. 7(a) for $V = 5$, where it is visible for times $t \lesssim 0.5$. Bearing in mind that the predictions of Calabrese and Cardy are based on an initial state with a finite correlation length, this finding is somewhat surprising. The slow algebraic decay of the correlations in the initial state causes the central site and the site at the boundaries of the system to already be correlated at $t = 0$. Therefore, the propagation of entangled quasiparticles created by the quench at the open boundary of the system leads to an additional signal which appears at decreasing values of $| i - j |$ when t is increased. This can be seen in Fig. 7(a) for sufficiently small values of $|i-j| \lesssim 10$ for times $1 \lesssim t \lesssim 2$, where an additional light cone moves towards the left at a velocity approximately half of the velocity of the main light cone. This observation is in agreement with the picture of Calabrese and Cardy applied to a semi-infinite chain: the left-moving signal in $C_{i,j}(t)$ is caused by the ballistic propagation of a pair of quasiparticles created during the quench, one of which is reflected at the open chain boundary.

We now turn to quenches deep into the CDW regime. In Fig. 7(b), our results for the quench from $V_0 = 0.5$ to $V = 40$ are displayed. It is not possible to identify the signature of a light cone in the data. Here $C_{i,j}(t)$ does not change significantly for times $t \gtrsim 1/t_{\mathrm{h}} = 1$, the typical time scale of a single fermion hopping process. In contrast to the previous cases, phase slips, marked by a reversal of the phase of the alternation of the correlations, are present. These phase slips are pronounced in $C_{i,j}(t)$

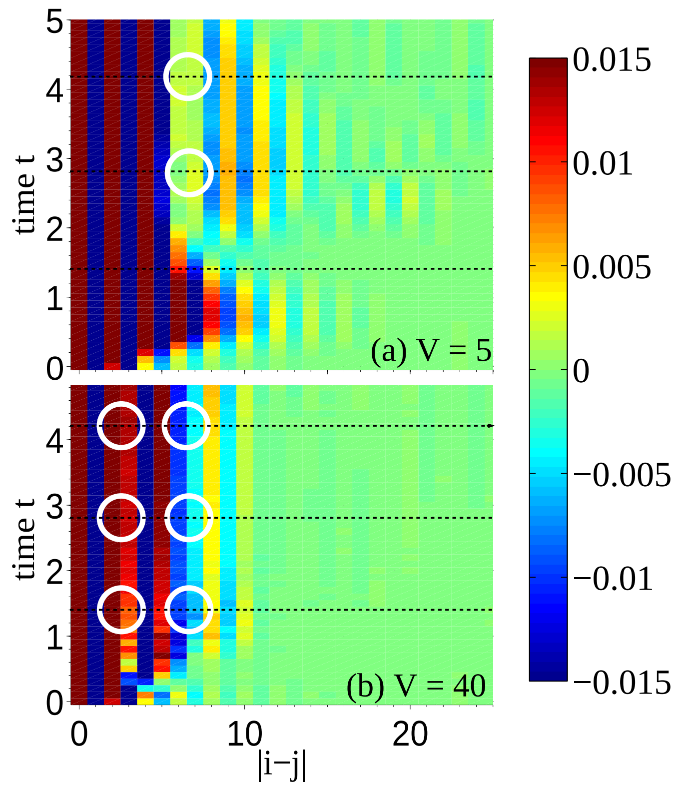

FIG. 7: (color online) Time evolution of the equal-time density correlation function $C_{i,j}(t)$ after a quench from the Luttinger-liquid ground state of $H(V_0)$ with $V_0 = 0.5$, evolved by the Hamiltonian $H(V)$, with (a) $V = 5$ and (b) $V = 40$. The dashed horizontal lines indicate the time steps shown in Figs. 8(b)-(d); the white circles highlight the phase slips visible there.

for $V = 40$, but can also be seen for $V = 5$ at distances $| i - j |= 6 - 7$ for times $t \gtrsim 2$. This can be seen in more detail in Fig. 8, where the phase slips for $V = 40$ are clearly visible in Figs. 8(b)-(d) (corresponding to times $t > 1.4$), while for $V = 5$ they only appear in Figs. 8(c) and (d).

The presence of such phase slips in the correlation function is not observed in the quenches starting from a CDW state which we discussed above, and suggests the presence of different CDW domains in the local density distribution which are separated by domain walls. Therefore, we now analyze the time evolution of the local density $\langle n_i \rangle(t)$ directly. Results for the local density distribution at $t = 5$ are shown for both $V = 5$ and $V = 40$ in Fig. 9. We find that domain walls (kinks) which separate different regions of CDW modulations and are stable in time are formed in the vicinity of the boundaries. Comparing the two values of V, we also find that the number of such domain walls is larger for larger values of V (at $t = 5$ we identify 2 such kinks for $V = 5$, and 4 for $V = 40$). Note that the density $\langle n_i \rangle(t) \approx 0.5$ in the middle of the chain shows almost no oscillation for this exactly half-filled lattice ($L = 50$, $N = 25$). The open ends of the

chain act as effective impurities and induce Friedel-like oscillations in the local density, which fall off similarly to the correlation function. Therefore, the stability of the domains in $\langle n_i\rangle(t) \approx 0.5$ reflects the behavior of the phase slips in the density correlation function. The presence of the phase slips and domains is reminiscent of the scenario proposed by Kibble[5] and Zurek[6,7] concerning the generation of topological defects in quenches that cross a quantum phase transition point[28] to a regime with spontaneous symmetry breaking. In the corresponding Kibble-Zurek mechanism, the duration of the quench τ is usually assumed to be finite. The characteristic critical scaling behavior near the crossed critical point leads to a power-law scaling of the defect density with $1/\tau$. In the current situation, the quench is sudden, corresponding to $\tau = 0$. The generation of defects in our system is therefore well beyond the Kibble-Zurek scaling regime. Pellegrini *et al.* have studied the scaling of the energy excess with duration in an XXZ spin chain undergoing a quench through the critical point.[29] It would be interesting to study directly the dependence of the defect density on the duration of the quench for this model in order to link our results for the instantaneous quench with the scaling prediction of the Kibble-Zurek mechanism.

C. Entanglement entropy of subsystems

In the previous sections we have characterized the propagation of information after a quench by the behavior of the equal-time density correlation function. We find two different scenarios: *(i)* information spreads ballistically through the system and leads to a light cone, and *(ii)* domain walls form after a quench deep into the symmetry broken regime, reminiscent of the Kibble-Zurek scenario.

The specific quantum nature of the problem under consideration can be further analyzed by directly investigating the entanglement buildup and spread in the system after the quench. A direct measure of the entanglement inside the system is given by the von Neumann entropy S_l of subsystems of size l as defined in Eq. (3). In Fig. 10, we show t-DMRG results for the time evolution $S_l(t)$ for some of the quenches considered previously. In all cases, we find that the block entropy for a particular subsystem size eventually saturates with time, to a final value that increases linearly with subsystem size. The initial phase of the time evolution of $S_l(t)$, however, shows characteristic differences for the two cases considered above: For those cases in which a distinct light cone has been observed in the density correlation function, the von Neumann entropy initially grows linearly, as seen in Fig. 10 (a) and (b) for quenches from $V_0 = 10$ to $V = 0$ and $V = 2$, respectively. There is therefore a characteristic time t_l at which the entropy saturates. One finds, to a good approximation, that t_l increases linearly with the subsystem length l, giving rise to a characteristic entropy propagation velocity that is roughly equal to the

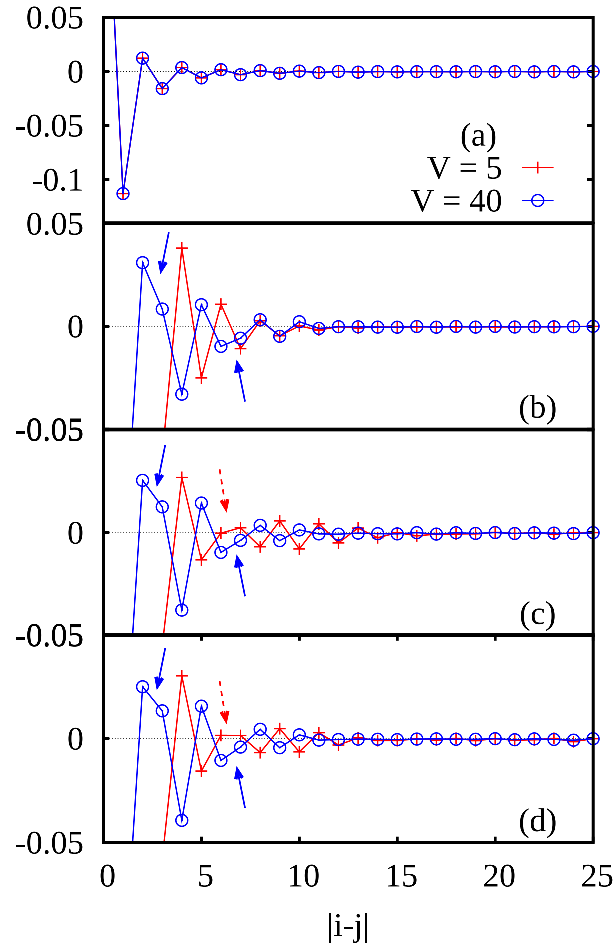

FIG. 8: (color online) Equal-time density correlation function $C_{i,j}(t)$ of spinless fermions after a quench from the Luttinger liquid ground state of $H(V_0)$ with $V_0 = 0.5$, evolved by the Hamiltonian $H(V)$, with $V = 5$ and $V = 40$, at fixed times $t = 0$ (a), $t = 1.4$ (b), $t = 2.8$ (c), and $t = 4.2$ (d) as functions of the separation $|i - j|$. The blue (solid) arrows highlight the position of the phase slips obtained for $V = 40$, the red (dashed) ones point to the phase slips for the case $V = 5$.

light-cone velocity u extracted from the density correlation function. The evolution of $S_l(t)$ in Fig. 10(c), corresponding to a quench from $V_0 = 10$ to $V = 5$, shows more complex initial behavior, with the formation of intermediate plateaux in $S_l(t)$, e.g., for $l = 5$, and the presence of local minima (dips) in $S_l(t)$, e.g., for $l = 3$. Turning now to Fig. 10(d), a quench from a LL initial state with $V_0 = 0.5$ to a point deep in the symmetry-broken regime, $V = 40$, we find a broad sublinear increase rather than a linear growth in $S_l(t)$, which, in addition, shows pronounced oscillations that damp out towards saturation. Similar oscillations were observed in Ref. 30 and were found to be due to the existence of additional local maxima of the group velocity of the quasiparticles. Thus, the behavior of $S_l(t)$ further corroborates the qualita-

tive difference between the cases with ballistic transport, showing a clear light-cone effect in the density correlation function $C_{i,j}(t)$, and cases with no distinct light cone. An interesting topic for future studies would be to analyze how the presence of kinks and phase slips in the local density and the density correlation function relates to this peculiar sublinear rise of the von Neumann entropy in more detail, and to understand the origin of the pronounced initial oscillations.

IV. SUMMARY AND DISCUSSION

In this work, we have studied the time evolution of correlation functions in a one-dimensional, half-filled system of spinless fermions with nearest-neighbor repulsive interactions. We perturb the system by changing the strength of the repulsion suddenly, i.e., we quench the interaction parameter and study the evolution of the density correlations at short to intermediate times. The investigations were carried out using the adaptive time-dependent density matrix renormalization group (t-DMRG). Since the half-filled system has a gapless, metallic (Luttinger-liquid) ground-state phase for small interaction strength and a gapped charge-density-wave insulating phase for large interaction strength, separated by a quantum phase transition point, it is possible to carry out quenches both within the two qualitatively very different phases and from one phase to another. When we do this, we find that quenches starting from the CDW insulating phase, either staying within the phase or quenching to the metallic phase, are both characterized by a light cone in the density correlation function, i.e., a horizon propagating with constant velocity, beyond which changes at a particular separation become visible. For the metallic case, the behavior agrees well with predictions based on conformal field theory made by Calabrese and Cardy,[14,15] although there appear to be small corrections to a horizon veloc-

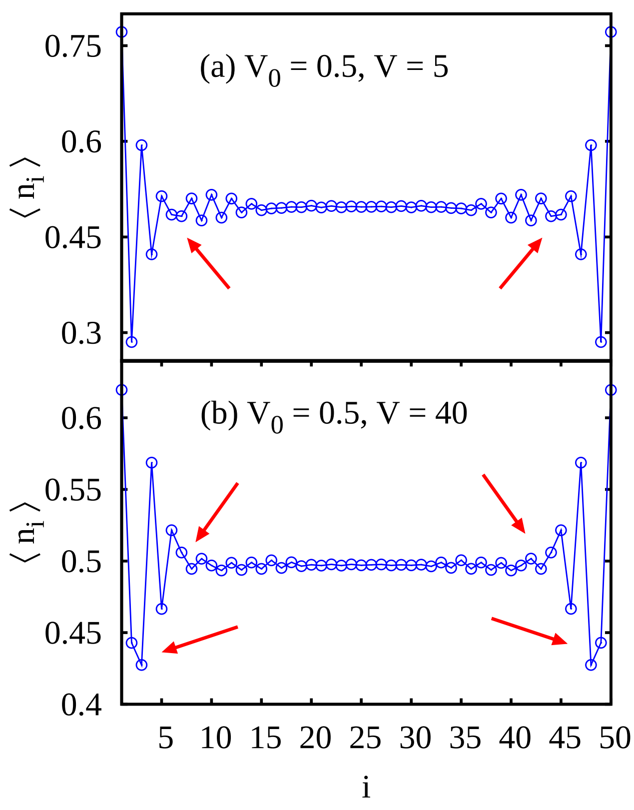

FIG. 9: (color online) Local density $\langle n_i\rangle(t)$ for a system of length $L = 50$ with $N = 25$ particles at time $t = 5$ after a quench from the Luttinger liquid ground state of $H(V_0)$ with $V_0 = 0.5$, evolved by the Hamiltonian $H(V)$, with (a) $V = 4$ and (b) $V = 40$. Arrows indicate the location of domain walls (kinks) in the local CDW pattern.

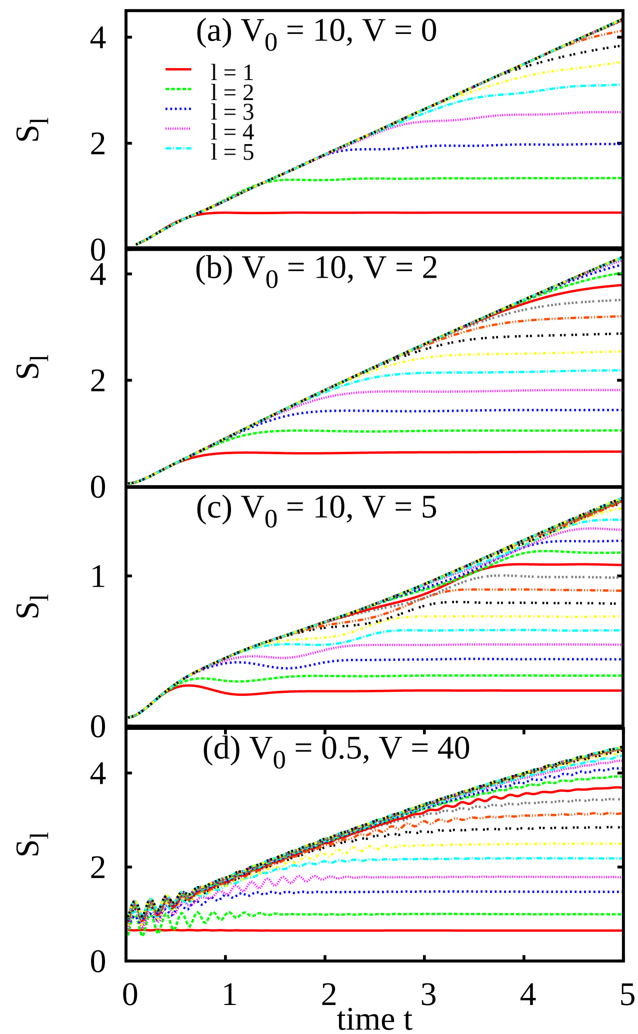

FIG. 10: (color online) Time evolution of the von Neumann entropy $S_l(t)$ for subsystems of different sizes $l = 1, 2, \ldots, 25$, after quenches with (a) $V_0 = 10 \to V = 0$, (b) $V_0 = 10 \to V = 2$, (c) $V_0 = 10 \to V = 5$, and (d) $V_0 = 0.5 \to V = 40$. The lowest line corresponds to $l = 1$, the next to $l = 2$ and so on; for clarity, the curves for $l \leq 5$ are labeled.

ity obtained from the exactly known velocity of charge excitations.

For quenches within the insulating phase, the presence of such a clear horizon for all values of the final interaction strength is a surprise. One might expect a horizon to occur when the energy scale of the gap is smaller than the excess energy (relative to the ground state) associated with the quench, but this does not seem to be the case. While there is presumably some sort of ballistically propagating quasiparticle associated with the horizon, its nature is not yet clear. In addition, the quench within the insulator is characterized by a density correlation function which remains exponentially decaying, but with a correlation length which grows in time, at least to the time scales that could be reached numerically. The behavior of the correlation length with time at longer times remains to be clarified.

For quenches from a metallic initial state to an insulating parameter value, the presence or absence of the remnants of a light cone depend on the strength of the final interaction. For relatively weak final interaction, a light cone is present to a time scale characterized by the propagation time to the end of the chain. Counter-propagating light cones with one-half the velocity of the initial light cone are also present. The counter-propagating features can be understood in terms of quasiparticles propagating in one direction only from the open ends of the chain. Examination of the correlation function reveals phase slips in the correlation functions and the formation of domains in the local density, which are indications of frozen-in domains generated by the quench. Such domains are predicted by the Kibble-Zurek mechanism,[5,6,7] which seems to be applicable here even though our sudden quench falls outside its scaling regime.

The behavior of the quantum information (von Neumann) entropy of subsystems supports the picture obtained from the behavior of the correlation functions. For cases where a light cone with a constant velocity is present in the correlation functions, there is a linear growth of the entropy with time up to a saturation value associated with the system size. A velocity associated with the saturation time is consistent with the light-cone velocity. For the cases with no light cone, the growth of the entropy with time is sublinear, with no well-defined saturation time. In addition, oscillatory behavior, which could be interesting to understand in more detail, occurs.

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